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Multidimensional solitons in complex media with variable dispersion: structure and evolution[1]

V.Yu. Belashov, A.V. Anoshen
Kazan State Power Engineering University, 51 Krasnosel'skaya, Kazan 420066 Russia

The problem of dynamics the multidimensional solitons in complex media with the dispersion is studied numerically. The applications to physics of the FMS waves in a magnetized plasma, and the 2-dimensional surface waves on shallow water are discussed.

In this paper we consider the problem of dynamics the multidimensional solitons which are described by the Kadomtsev-Petviashvili (KP) equation

$$\partial_t u + \alpha u \partial_x u + \beta \partial_x^3 u = \kappa \int_{-\infty}^{x} \Delta_\perp u\, dx, \qquad (1)$$

in complex media with the varying in time and/or space dispersive parameter $\beta = \beta(t,\mathbf{r})$. This problem is mainly interesting from the point of view of its evident applications in physics of the concrete complex media with the dispersion. For example, such situation may have place in the problems of the propagation of the 2D gravity and gravity-capillar waves on the surface of "shallow" water [1] when coefficient β is defined respectively as $\beta = c_0 H^2/6$ and $\beta = (c_0/6)\left[H^2 - 3\sigma/\rho g\right]$ where H is the depth, ρ is the density, and σ is the coefficient of surface tension of fluid. If $H = H(t,x,y)$ the dispersive parameter also becomes the function of the coordinates and time. Similar situation may have place on studying of the evolution of the 3D FMS waves in magnetized plasma in case of the inhomogeneous and/or non-stationary plasma and magnetic field [2] when β is function of the Alfvén velocity $v_A = f[B(t,\mathbf{r}), n(t,\mathbf{r})]$ (n is the plasma density) and an angle $\theta = (\mathbf{k}^\wedge\mathbf{B})$: $\beta = v_A(c^2/2\omega_{0i}^2)(\cot^2\theta - m/M)$. It is well known that the 1D solutions of the KdV equation with $\beta = \text{const}$ in dependence on value of the dispersion parameter are divided into two classes: at $|\beta| < u_0(0,x)l/12$ (l is the characteristic wave length of the initial disturbance) they have soliton character, in a return case – they are the wave packets with asymptotes being proportional to the derivative of the Airy function. In this cases the KdV equation can be integrated analytically by the IST method. But, if $\beta = \beta(x,t)$ such approach is impossible principally, and it is necessary to resort to a numerical simulation in the conforming problems. Similar situation has place in a non-one-dimensional model described by the KP equation: if the analytical solutions of the KP equation are known that in case $\beta = \beta(t,\mathbf{r})$ the dispersion term of equation becomes quasi-linear and the model being not exactly integrable [3].

In this paper the results of numerical experiments on study of structure and evolution of the nonlinear waves described by the KP equation with $\beta = \beta(t,\mathbf{r})$ are considered distracting from a concrete type of media. The numerical experiments were conducted for several model types of function β when at $t < t_{cr}$ $\beta = \beta_0 = \text{const}$, and at $t \geq t_{cr}$

$$1)\ \beta(x) = \begin{cases} \beta_0, & x \le a; \\ \beta_0 + c, & x > a; \end{cases} \qquad (2)$$

$$2)\ \beta(x,t) = \begin{cases} \beta_0, & x \le a; \\ \beta_0 + nc, & n = (t - t_{cr})/\tau = 1,2,\ldots;\ x > a; \end{cases} \qquad (3)$$

$$3)\ \beta(t) = \beta_0\left(1 + k_0\bar\beta \sin\omega t\right),\ \bar\beta = (\beta_{\max} - \beta_{\min})/2, \qquad (4)$$
$$0 < k_0 < 1, \quad \pi/2\tau < \omega < 2\pi/\tau,$$

a and c are constants. In terms of the propagation of the waves on the shallow water that means that after reaching t_{cr}, respectively: 1) sharp "break of bottom"; 2) gradual "change of a height" of a segment of bottom; and 3) the "oscillations of bottom" with time are happen.

The results obtained in the numerical experiments on evolution of the 2D solitons described by the KP equation model (1) with $\beta = \beta(t,\mathbf{r})$ enable to obtain the different types of both stable and unstable solutions including the solutions of the mixed "soliton – non-soliton" type for different character of the dispersion variations in time and space. The obtained results open new perspectives in investigation of a number of applied problems of dynamics of the multidimensional nonlinear waves in concrete complex media with the dispersion.

References

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[3] V.Yu.Belashov, S.V. Vladimirov, *Solitary Waves in Dispersive Complex Media*, Springer-Verlag GmbH & Co.KG, 2003.

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